Chapter 4
Polynomial Functions

1. **Graphing Polynomial Functions**
2. Adding, Subtracting, and Multiplying Polynomials
3. Dividing Polynomials
4. Factoring Polynomials
5. Solving Polynomial Equations
6. The Fundamental Theorem of Algebra
7. Transformations of Polynomial Functions
8. Analyzing Graphs of Polynomial Functions
9. Modeling with Polynomial Functions

4.1 - Graphing Polynomial Functions

Warm-up - Identify graphs with functions

a. $f(x) = x^3 - x$ **A**

b. $f(x) = -x^3 + x$ **D**

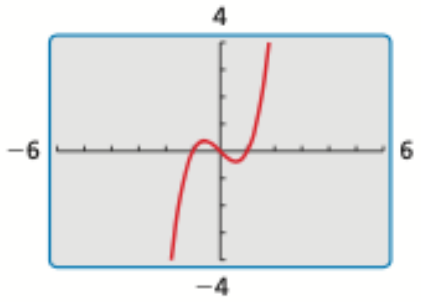
c. $f(x) = -x^4 + 1$ **B**

d. $f(x) = x^4$ **F**

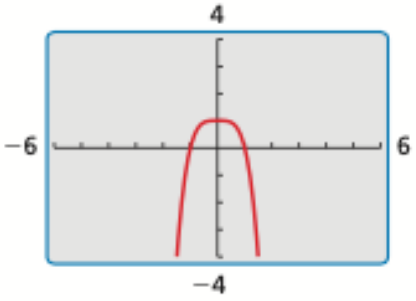
e. $f(x) = x^3$ **E**

f. $f(x) = x^4 - x^2$ **C**

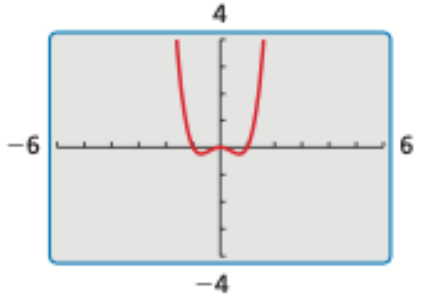
A.



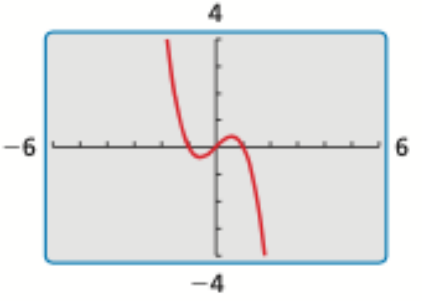
B.



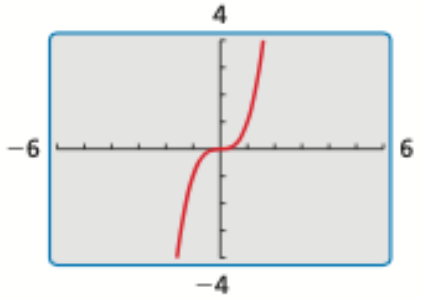
C.



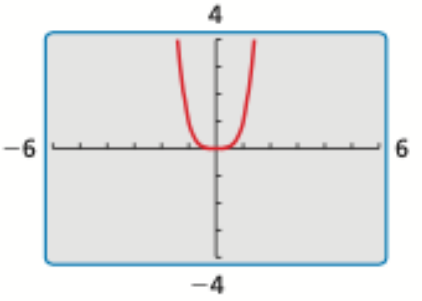
D.



E.



F.



4.1 - Graphing Polynomial Functions

Polynomial Function

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$$

where:

- $a_n \neq 0$
- the exponents are all whole numbers
- a_n is the leading coefficient
- n is the degree
- a_0 is the constant term

4.1 - Graphing Polynomial Functions

Polynomial Function

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$$

Common Polynomial Functions			
Degree	**Type**	**Standard Form**	**Example**
0	Constant	$f(x) = a_0$	$f(x) = -14$
1	Linear	$f(x) = a_1 x + a_0$	$f(x) = 5x - 7$
2	Quadratic	$f(x) = a_2 x^2 + a_1 x + a_0$	$f(x) = 2x^2 + x - 9$
3	Cubic	$f(x) = a_3 x^3 + a_2 x^2 + a_1 x + a_0$	$f(x) = x^3 - x^2 + 3x$
4	Quartic	$f(x) = a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0$	$f(x) = x^4 + 2x - 1$

4.1 - Graphing Polynomial Functions

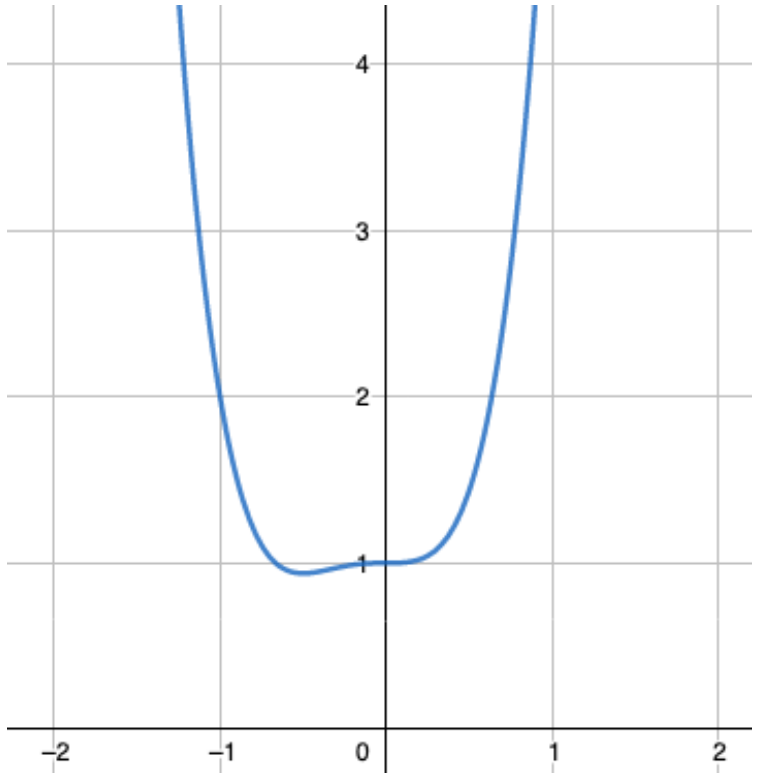
Leading coefficient

$$f(x) = 3x^4 + 2x^3 + 1$$

How do we justify end behavior?

$$\lim_{x \to +\infty} f(x) = +\infty$$

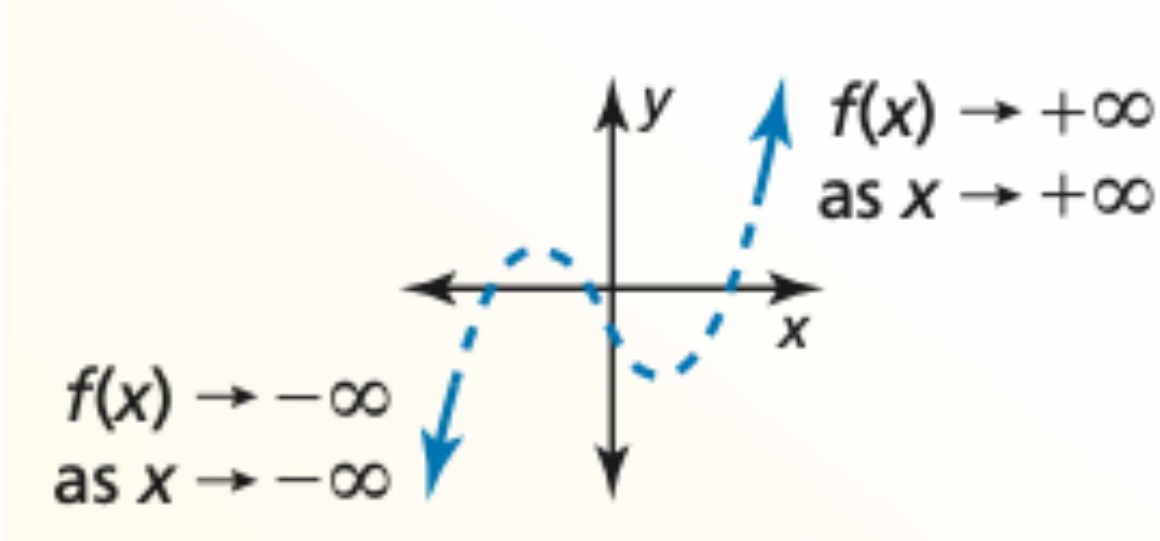
$$\lim_{x \to -\infty} f(x) = +\infty$$



4.1 - Graphing Polynomial Functions

End Behavior of a function is the behavior of the graph as x approaches positive infinity (+∞) or negative infinity (-∞).

The end behavior is determined by the function's degree and the sign of its leading coefficient.



4.1 - Graphing Polynomial Functions

End behavior

$$g(x) = 2x^3 - 3x^2 + x - 1$$

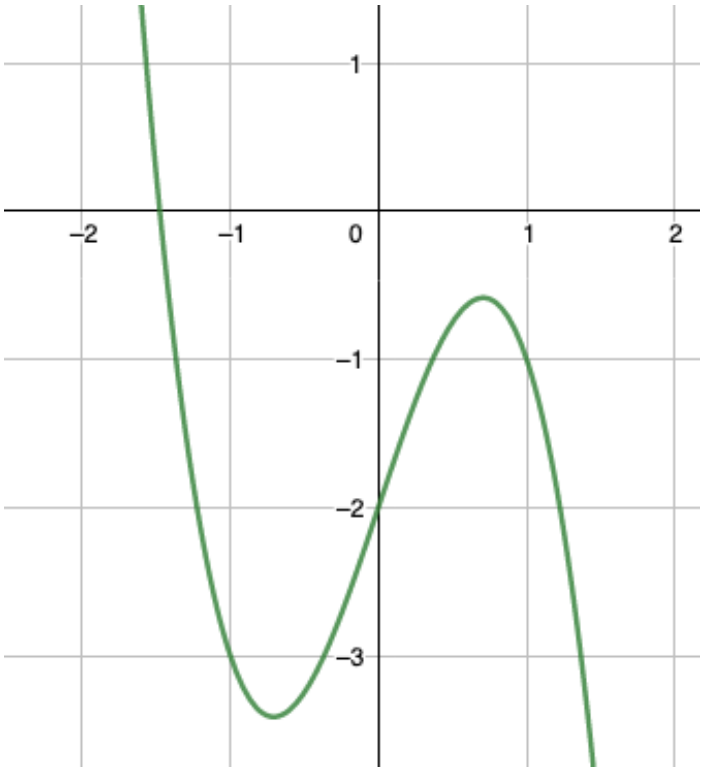
$$\lim_{x \to +\infty} g(x) = +\infty$$

$$\lim_{x \to -\infty} g(x) = -\infty$$

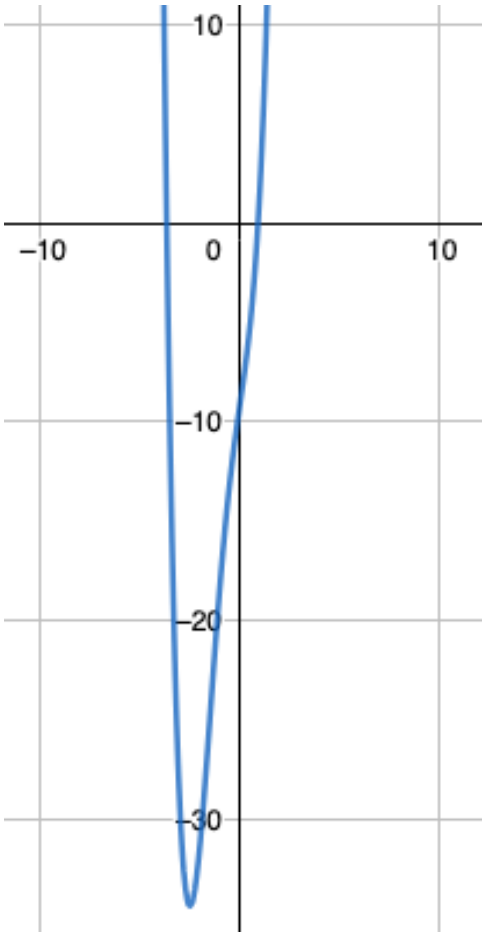
4.1 - Graphing Polynomial Functions

Practice - Determine end behavior for equation

1) $f(x) = -2x^3 + 3x - 2$



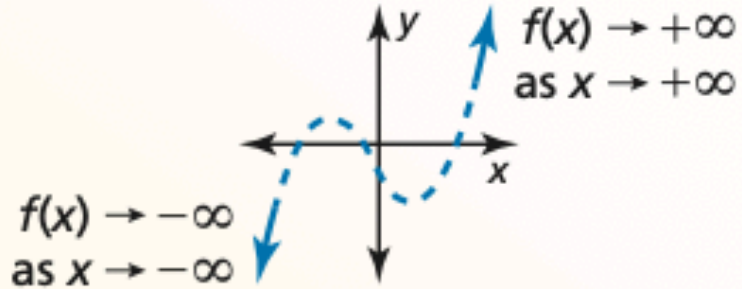
2) $f(x) = x^4 + 3x^3 + 7x - 9$



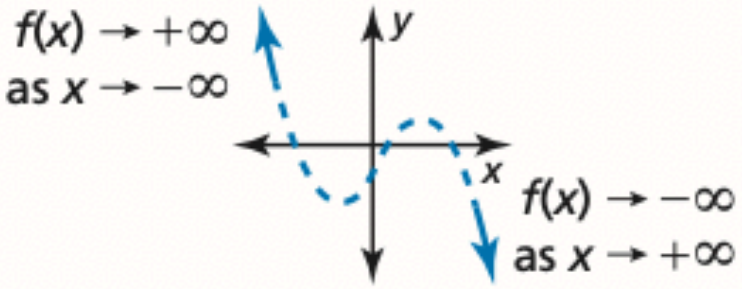
4.1 - Graphing Polynomial Functions

End Behavior of Polynomial Functions

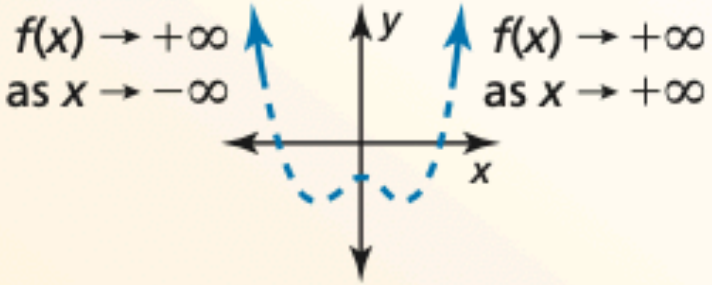
Degree: odd
Leading coefficient: positive



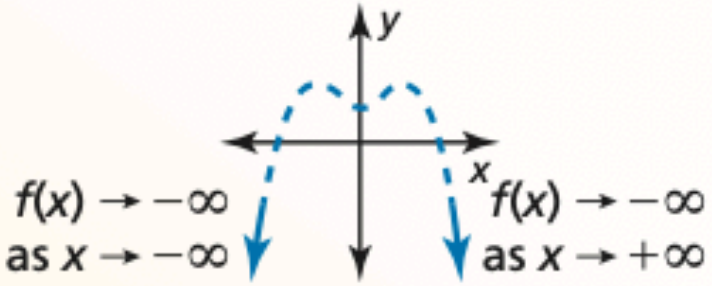
Degree: odd
Leading coefficient: negative



Degree: even
Leading coefficient: positive



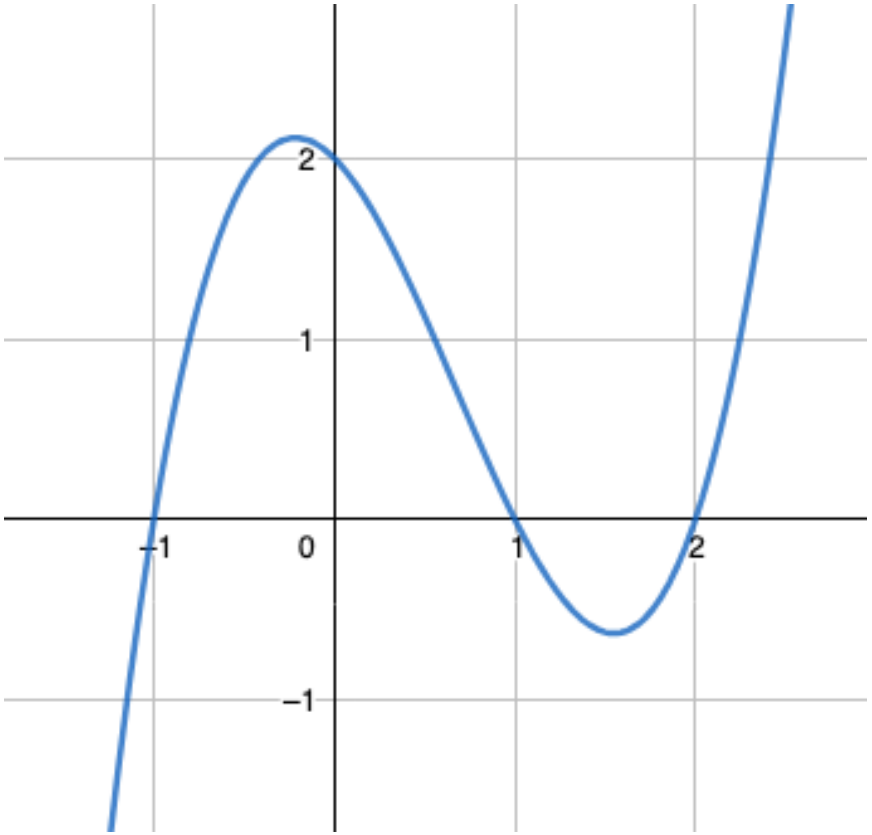
Degree: even
Leading coefficient: negative



4.1 - Graphing Polynomial Functions

Graph $f(x) = x^3 - 2x^2 - x + 2$

$= (x + 1)(x - 1)(x - 2)$



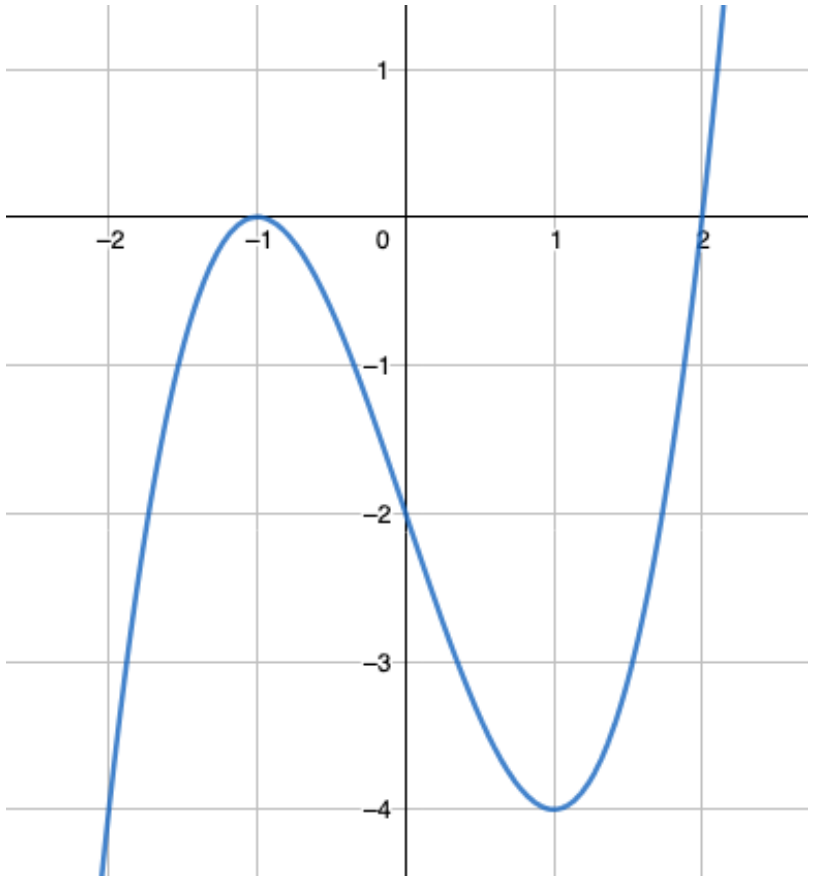
4.1 - Graphing Polynomial Functions

Graph

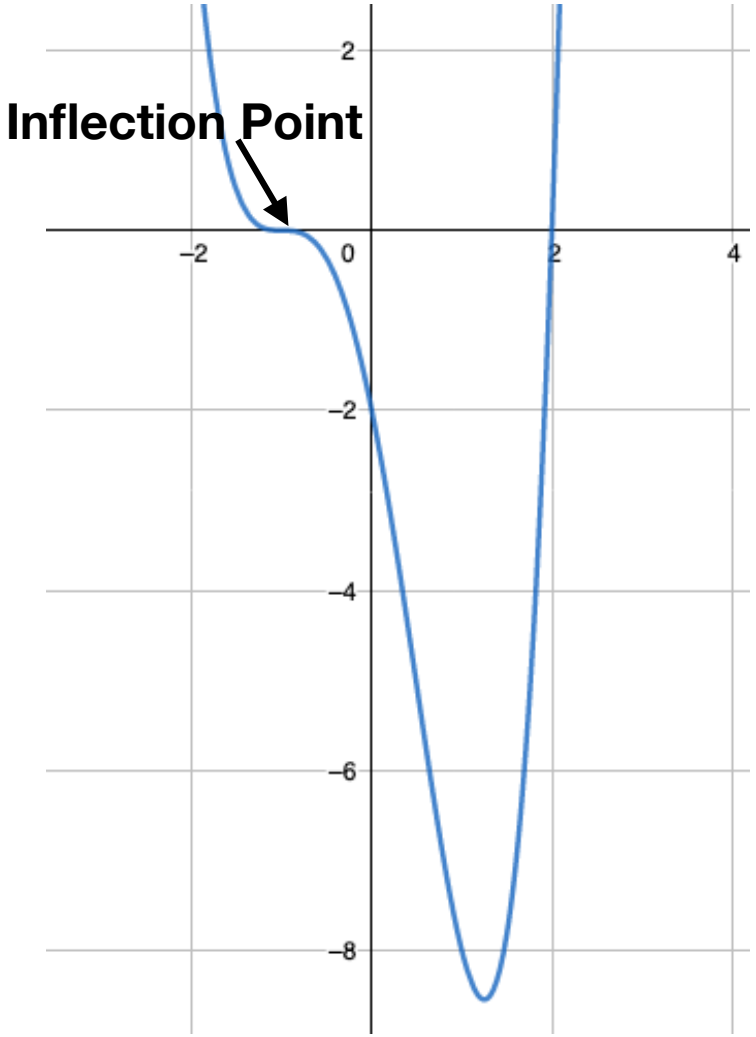
$f(x) = (x+1)^2(x-2)$

Multiplicity of -1?

Multiplicity of 2?



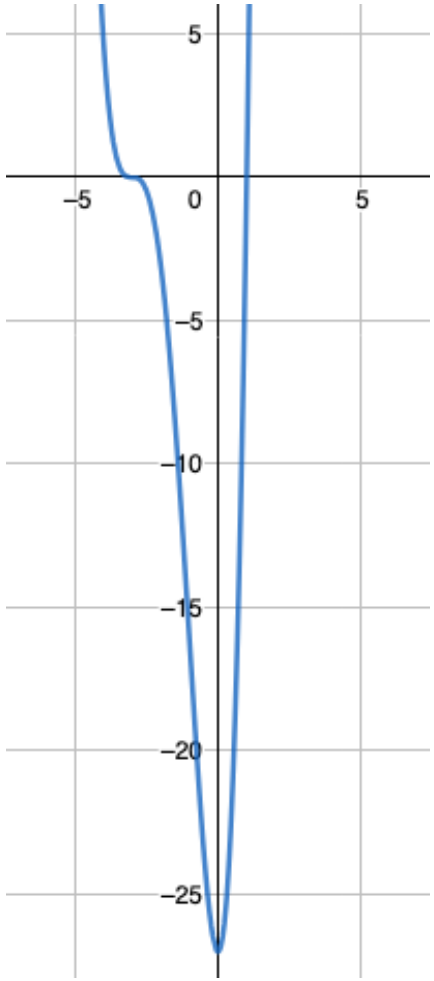
$f(x) = (x+1)^3(x-2)$



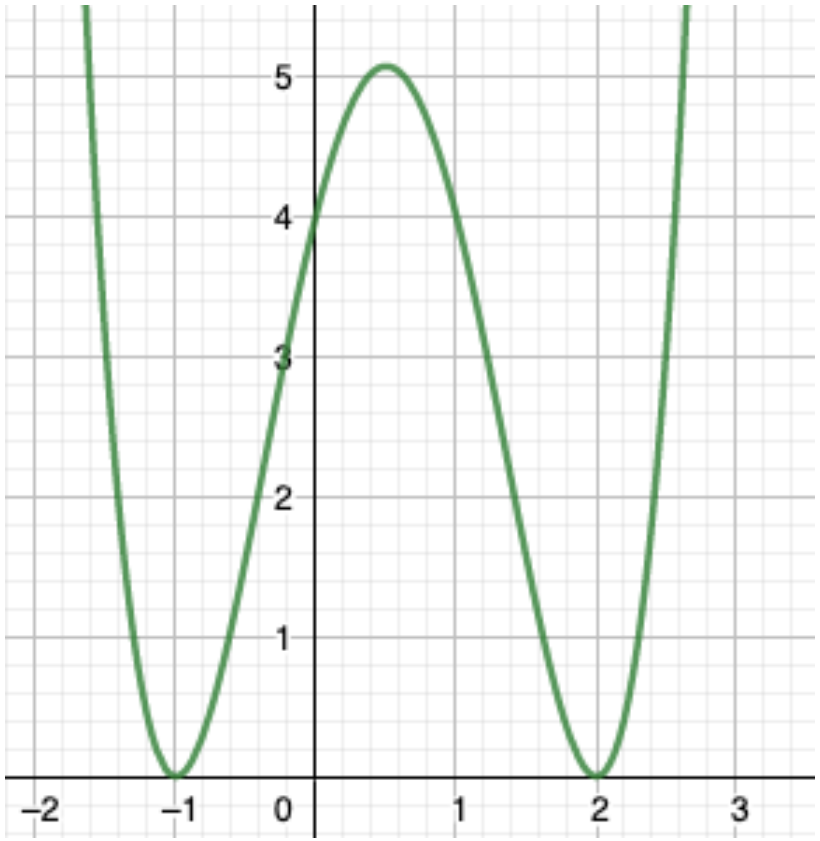
4.1 - Graphing Polynomial Functions

Practice - Graph

1) $f(x) = (x+3)(x-1)(x+3)^2$



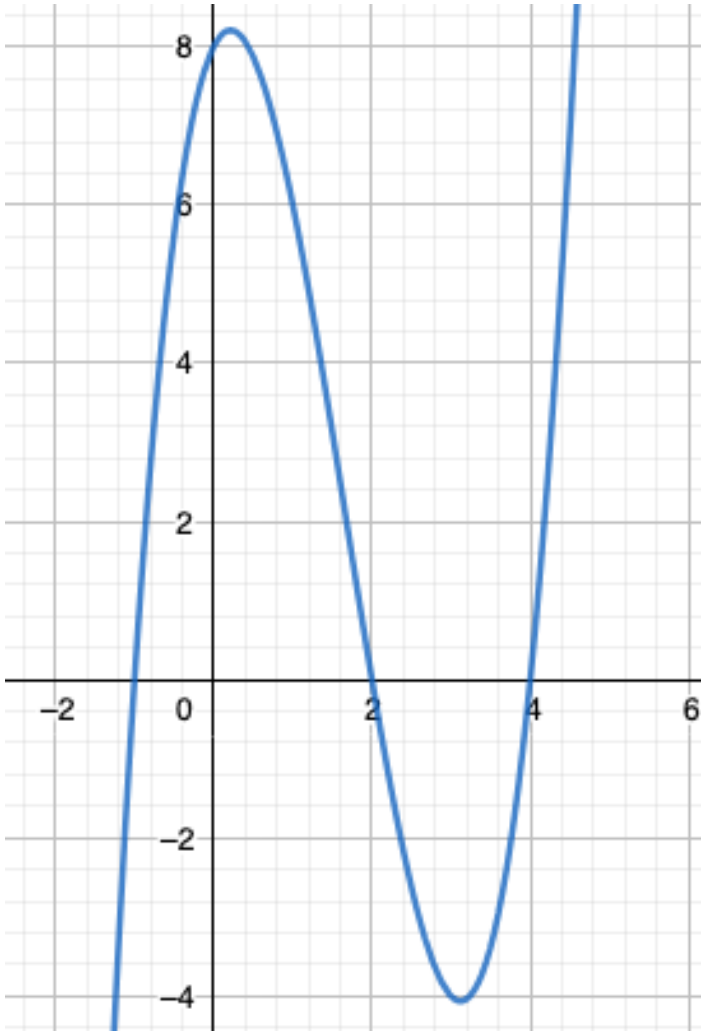
2) $f(x) = (x^2+2x+1)(x^2-4x+4)$



4.1 - Graphing Polynomial Functions

Find the cubic function where -1, 2, and 4 are roots.

$$f(x) = (x+1)(x-2)(x-4)$$
$$= (x^2 - x - 2)(x-4)$$
$$= x^3 - x^2 - 2x - 4x^2 + 4x + 8$$
$$f(x) = x^3 - 5x^2 + 2x + 8$$



4.1 - Graphing Polynomial Functions

Practice - Find the cubic function with the given roots.

1) -3, 1, 2

$f(x) = x^3 - 7x + 6$

2) $1 - \sqrt{3}, 1 + \sqrt{3}$, -2

$f(x) = x^3 - 6x - 4$

4.1 - Graphing Polynomial Functions

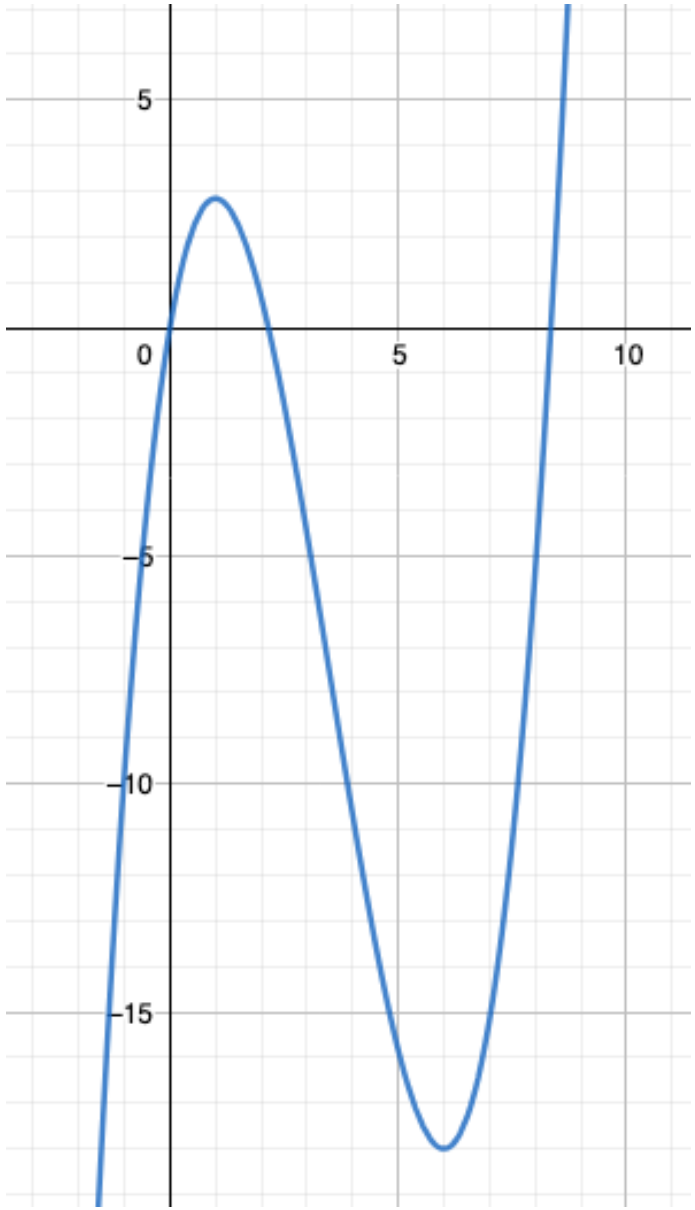
Graph

$$f(x) = -(x-3)^2(x+5)(x-11)^3$$

4.1 - Graphing Polynomial Functions

Graph the following

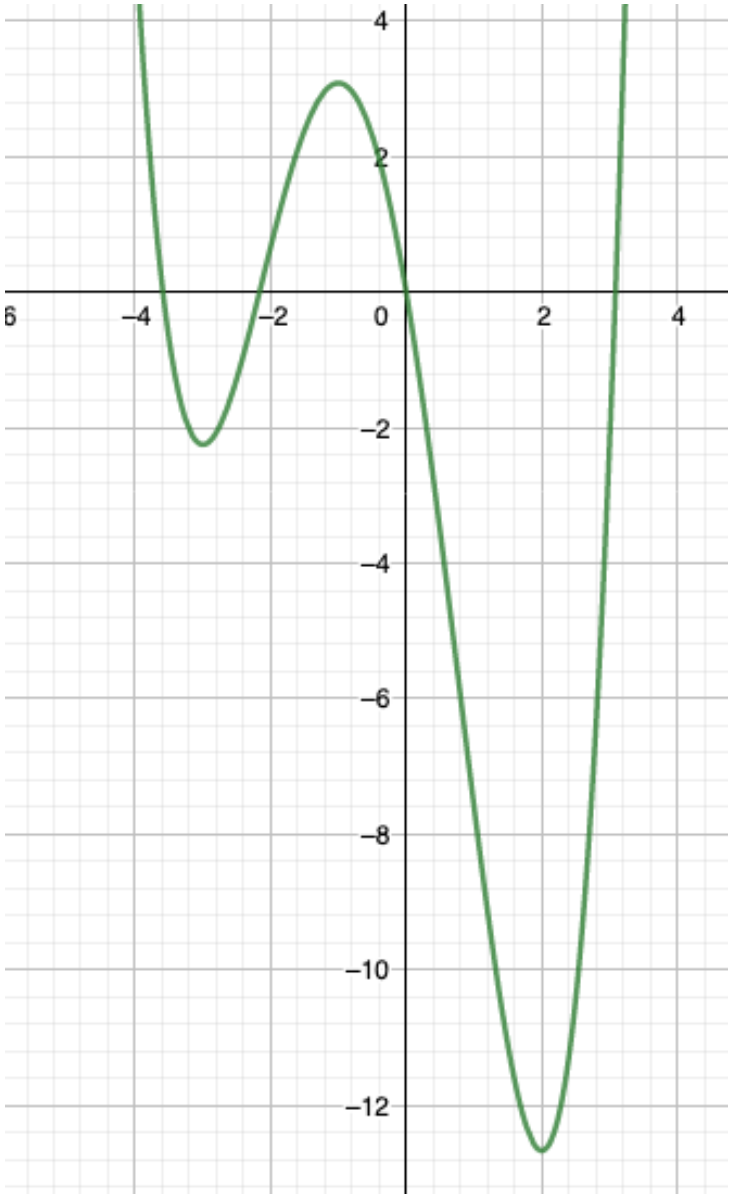
y increasing $x < 1$
y decreasing $1 < x < 6$
y increasing $x > 6$



4.1 - Graphing Polynomial Functions

Graph the following

y decreasing $x < -3$
y increasing $-3 < x < -1$
y decreasing $-1 < x < 2$
y increasing $x > 2$



4.1 - Graphing Polynomial Functions

Graph

$$f(x) = |(x+5)(x+2)(x-1)(x-3)^2|$$